Department of Mathematics

Course Profile

Course Number: MATH 323	**Course Title: Calculus of Variations**
Required / Elective: Elective	**Prerequisites:** None
Catalog Description: Historical approach to basic problems; variation of a functional; weak and strong extrema; Euler-Lagrange equations; variational derivative, higher order derivatives, subsidiary conditions; variable end point problems; broken extremals. Noether's heorem, Hamilton- Jacobi Equation, Jacobi's theorem; quadratic functionals, second variation of a functional. Direct methods, Ritz and Kantorovich methods.	**Textbook / Required Material:** Textbook: I.M.GELFAND & S.V. FOMIN, *Calculus of Variation,* Prentice Hall, 1963.

Course Structure / Schedule: (3+0+0) 3/ 7 ECTS

Elements of the theory: Functionals. Function spaces. Variation of a functional. Several variables. Euler Equation. Variable end point problems. Variational derivative. Invariance of Euler's Equation. Examples. ***Further Generalizations***: Fixed end point problems. Parametric form. Higher order derivatives. Subsidiary conditions. ***The General Variation of a Functional***: Basic Formula. Moving end points. Broken Extremals. Weierstrass-Erdmann Conditions. Examples. ***The Canonical Form of Euler Equations and Related Topics***: Canonical form of EE. First Integrals of EE. Legendre Transformations. Examples. Canonical Transformations. Noether's Theorem. Principle of least action. Conservation Laws. Examples. Hamilton-Jacobi Equation. Jacobi's Theorem. Examples. ***The Second Variation. Sufficient Conditions for Weak Extremum***: Quadratic functionals, second variation of a functional. Legendre's condition. ***Fields. Sufficient Conditions for Strong Extremum***: Definitions. Field of functionals. Hilbert's invariant integrals. Strong extremum. Examples. ***Direct Methods in the Calculus of Variations*****:** Minimizing sequences. Method of finite difference. Ritz method. Examples. The Sturm-Liouville problems. General review, more examples. ***Variational Problems involving Multiple Integrals***: Variation of functionals on a fixed region. Continuous mechanical systems. Variation of a functional on a variable region. Applications to field theory. ***Direct Methods in the Calculus of Variations*****:** Minimizing Sequences. Method of finite difference. Ritz method. Examples. The Sturm-Liouville Problems. General review, more examples.

Design content: None	**Computer usage:** Partly

Course Outcomes: By the end of the course the students should be able to:

1. give a modern treatment of the calculus of variations from a rigorous perspective, blending classical and modern approaches and applications. **[2,3, 6],**
2. learn rigorous results in the classical and modern calculus of variations and see possible behaviour and application of these results in examples. **[3, 6].**

[2] demonstrate knowledge of mathematics to construct, analyze and interpret mathematical models,
[3] demonstrate the ability to apply mathematics to the solutions of problems,
[6] have a basic knowledge of the main fields of mathematics, including analysis,

algebra, differential equations, differential geometry.

Recommended reading:

1. U. Brechtken-Manderscheid, *Introduction to the Calculus of Variations* (Chapman & Hall, 1991).
2. H. Sagan, *Introduction to the Calculus of Variations* (Dover, 1992).
3. J. Troutman, *Variational Calculus and Optimal Control* (Springer–Verlag, 1995).

Teaching methods: Three hours theoretical presentation with illustrative problem solving.

Assessment methods:

Homework, quiz, midterm and final exams.

Student workload:

Pre-reading ....35 hrs

Lectures ……………………………………45 hrs

Preparatory reading ………………….. 35 hrs

Literature review for presentation………… 45 hrs

Team work for presentation ………………15 hrs

TOTAL ……………………………… 175 hrs …… to match 25x7 ECTS

Prepared by: Prof.Dr.Esin İnan	Revision Date: 08.02.2010