## **Department of Mathematics**

# **Course Profile**

Course Number: MATH 321	Course Title: Partial Differential Equations	
Required / Elective: Required	Prerequisite : None	
<b>Catalog Description:</b> Basic definitions, first-order partial differential equations; types of second-order partial differential equations. The wave equation, the diffusion equation; maximum principle. Boundary value problems, separation of variables. Laplace's equation, Green's identities and functions.	<b>Textbook / Required Material:</b> "Partial Differential Equations-An Introduction", by W.A. Strauss, Published by John Wiley & Sons, Inc., 1992.	

### Course Structure / Schedule : (3+0+0) 3 / 7 ECTS

#### **Extended Description:**

This is an introductory Partial Differential Equations course for students in mathematics. The focus will be on the analytical solution techniques for second-order linear partial differential equations such as wave equation, diffusion equation and Laplace equation. The course will cover most sections of the first seven chapters of the book. The first chapter discusses the meaning of a partial differential equation (first-order linear equations, flows, vibrations, and diffusions, initial and boundary conditions, well-posed problems, types of second-order equations). The second chapter is about the wave and diffusion equations on the whole real line (the wave equation, causality and energy, the diffusion equation, diffusion on the whole line, comparison of waves and diffusions). The third chapter considers the inhomogeneous wave and diffusion equations (diffusion with a source, waves with a source). The fourth chapter is about the wave and diffusion equations on a finite interval (separation of variables, the Dirichlet condition, the Neumann condition the Robin condition). The fifth chapter is about Fourier series (the coefficients of Fourier series, even, odd, periodic, and complex functions, orthogonality and general Fourier series, completeness, inhomogeneous boundary conditions). The sixth chapter is devoted to the Laplace equation (rectangles and cubes, Poisson's formula, circles, wedges and annuli). The seventh chapter is about the Green function (Green's first identity, Green's second identity, Green's functions, half-space and sphere).

Design content: None	<b>Computer usage:</b> No computer usage required.
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**Course Outcomes:** By the end of the course the students should be able to:

- 1. derive the transport, diffusion and wave equations in one spatial dimension [2,3,6],
- 2. classify a given second-order PDE as hyperbolic, parabolic or elliptic equation [2,3,6],
- 3. solve the initial-value problem for the wave equation [2,3,6],
- 4. prove the maximum principle for the diffusion equation [2,3,6],
- 5. demonstrate the uniqueness of the solution of an initial-value problem for the diffusion equation (by the maximum principle and by the energy method) [2,3,6],
- 6. solve the initial-value problem for the diffusion equation [2,3,6],

- 7. solve the initial-value problems for the inhomogeneous diffusion and inhomogeneous wave equations [2,3,6],
- 8. solve the homogeneous diffusion and homogeneous wave equations on a finite interval using the method of separation of variables (for the Dirichlet, Neumann and Robin conditions) [2,3,6],
- 9. find the coefficients in the Fourier sine series, the Fourier cosine series and the full Fourier series [3,6],
- 10. prove the maximum principle for the Laplace equation in two dimensions [2,3,6],
- 11. demonstrate the uniqueness of the Dirichlet problem for the Laplace equation using the maximum principle **[2,3,6]**,
- 12. solve the Laplace equation in a rectangle using the method of separation of variables [2,3,6],
- 13. Prove the Posisons formula for the Dirichlet problem for a circle [2,3,6],
- 14. solve the Laplace equation in a wedge, in an annulus and for the exterior of a circle using the method of separation of variables **[2,3,6]**,
- 15. know the notion of Green's function [2,3,6],
- 16. express the solution of a Dirichlet problem for the Laplace equation through the Green function **[2,3,6]**.

# [2] demonstrate knowledge of mathematics to construct, analyze and interpret mathematical models,

[3] demonstrate the ability to apply mathematics to the solutions of problems,[6] have a basic knowledge of the main fields of mathematics, including analysis, algebra, differential equations, differential geometry.

**Recommended reading** : "A First Course in Partial Differential Equations with Complex Variables and Transform Methods", by H.F. Weinberger, Published by Dover Publications, Inc., 1995.

"Partial Differential Equations-An Introduction", by E.C. Young, Published by Allyn&Bacon, 1972.

**Teaching methods:** 

Preparatory-readings, lectures, discussions, assignments

Assessment methods:

Midterm exams, final exam

#### Student workload:

Preparatory reading	54 hrs
Lectures	42 hrs
Assignments	56 hrs
Discussions	14 hrs
Midterm exams	6 hrs
Final exam	3 hrs

TOTAL ...... 175 hrs ... to match 25 x 7 ECTS

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